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THE RESONANCE FREQUENCIES OF SHELLS OSCILLATING IN AN INFINITE FLUID

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Solution of the problem of forced oscillations of a spherical shell immersed in an infinite compressible fluid is derived and analyzed on the assumption of a fairly rapidly changing stress state. It is shown that in spite of absence of real natural oscillation frequencies [1] of the system "shell-fluid", there exist frequencies that produce effects similar to resonance. Such frequencies were found to exist in the case of a particular pattern of pressure variation in the neighborhood of a spherical shell, when the basic contribution is provided by the damped pressure component. Formulas for the damped pressure component and resonance frequencies of the system are obtained in the case of high-frequency oscillations of the shell on the assumption that a similar pattern of fluid pressure is also possible in the neighborhood of an arbitrary convex shell (*).

1. Let us consider the steady oscillations of a closed shell in a compressible fluid. The oscillations which are assumed to be mainly flexural (quasitransverse), are defined by the equations of rapidly changing stress-strain state [2]

$$h_*^2\Delta_2^2 W - \Delta_1\chi - \lambda^2 W + p\,|_S - q = 0, \quad \Delta_2^2\chi + \Delta_1 W = 0 \tag{1.1}$$
$$\lambda^2 = \omega^2(\rho_0 / E)^2, \quad W = 2Ehw, \quad h_*^2 = h^2[3(1-\nu^2)]^{-1}$$

where ω is the oscillation frequency; w is the normal displacement of the shell; χ is a function of tangential displacement; h is the shell half-thickness; ρ_0, E, ν are, respectively, the shell material density, the modulus of elasticity, and the Poisson coefficient; Δ_2 and are the Laplace and Vlasov's operators on the shell surface S; q is the amplitude of periodic internal loading of the shell, and p is the acoustic pressure created in the fluid by the oscillating shell.

Pressure distribution in the fluid, after elimination of the transient component, conforms to the Helmholtz equation and the radiation condition

$$\Delta p + k^2 p = 0, \quad \lim_{x_3\to\infty} x_3\left(\frac{\partial p}{\partial x_3} + ikp\right) = 0 \tag{1.2}$$
$$b\frac{\partial p}{\partial x_3}\Big|_S = W, \quad b = \frac{2Eh}{\rho_f\omega^2}, \quad k = \frac{\omega}{c_f}$$

*) See A. P. Kachalov, The ray method for flexural oscillations of a shell immersed in fluid. Annals of Sci. Seminars of Leningrad Branch of the Steklov Mathem. Inst., Vol.62, 1976.

where ρ_f and c_f are, respectively, the fluid density and the speed of sound in it, x_3 is a coordinate orthogonal to surface S, and Δ is the Laplace operator in space.

In the case of a spherical shell $\Delta_2 = r_0\Delta_1$, $R_1 = R_2 = r_0$ and its is possible to separate the particular equation for W by eliminating the function X and, then, substitute the normal derivative of pressure at the shell surface for the deflection function. As a result, we obtain from the equation for the shell oscillation form the boundary condition for the Helmholtz equation in terms of pressure in the fluid, and thus reduce the problem to a purely acoustic one. In spherical coordinates it is of the form

$$\Delta p + k^2 p = 0, \quad \lim_{r\to\infty} r\,(\partial p / \partial r + ikp) = 0 \tag{1.3}$$
$$\Delta_2\,[b\,(h_*{}^2\Delta_2{}^2 - \lambda^2 + r_0{}^{-2})\partial p / \partial r + p - q]\,|_{r=r_0} = 0$$
$$W = b\partial p / \partial r\,|_{r=r_0}, \quad \Delta_2 W = -r_0\Delta_2{}^2 X$$

2. Separation of variables in problem (1.3) yields the following equations and boundary conditions:

$$p\,(r, \theta, \beta) = R\,(r) Y\,(\theta, \beta) \tag{2.1}$$
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[k^2 - \frac{n\,(n+1)}{r^2}\right] R = 0 \quad (n = 0, 1, 2, \ldots)$$
$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\beta^2} + n\,(n+1)\,Y = 0$$
$$\Delta_2\,\{[b\,(\lambda_{0n}^2 - \lambda^2)\,R' + R]\,Y - q\}\,|_{r=r_0} = 0, \quad \lim_{r\to\infty} r\,(R' + ikR) = 0$$
$$\lambda_{0n} = \omega_{0n} / c_0, \quad \omega_{0n} = c_0 r_0{}^{-1}\,[(h_* / r_0)^2 n^2\,(n+1)^2 + 1]^{1/2},$$
$$c_0 = \sqrt{E / \rho_0}$$

where ω_{0n} are frequencies of natural quasitransverse oscillations with considerable variation at the spherical shell out of contact with the fluid.

The Hankel spherical functions

$$h_n{}^{(2)}(x) = ({}^1/_2\pi / x)^{1/2}\,[J_q(x) - iN_q(x)], \quad x = kr, \ q = n + {}^1/_2$$
$$(n = 0, 1, 2, \ldots)$$

represent the integrals of Eq. (2.1) for $R\,(r)$ that satisfy the radiation conditions at infinity [3], and the spherical harmonics

$$Y_n{}^{(m)}(\theta, \beta) = P_n{}^{(m)}(\cos\theta) e^{im\beta}\ (m, \ n = 0, 1, 2, \ldots; \ m < n)$$

where $P_n{}^{(m)}$ are adjoint Legendre polynomials of power n and order m, are the integrals of the equation for $Y\,(\theta, \beta)$.

The general solution of Eq. (1.3) can be represented in the form of series

$$p\,(r, \theta, \beta) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{nm} h_n^{(2)}(kr)\, Y_n^{(m)}(\theta, \beta) \tag{2.2}$$

which contains unknown constants A_{nm} that are determined by the boundary conditions at the sphere surface. To determine the constants A_{nm} we represent the internal pressure $q\,(\theta, \beta)$ in the form of series in spherical functions (for this it is sufficient for $q(\theta, \beta)$ to be a twice differentiable function [4])

$$q(\theta, \beta) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} f_{nm} Y_n^{(m)}(\theta, \beta) \tag{2.3}$$

$$f_{nm} = \frac{2\pi\varepsilon_m}{2n+1} \frac{(n+m)!}{(n-m)!} \int_0^{2\pi} \int_0^{\pi} q Y_n^{(m)} \sin\theta \, d\theta \, d\beta, \quad \varepsilon_0 = 2, \quad \varepsilon_m = 1 \quad (m > 0)$$

For instance, in the case of concentrated normal force applied at point θ_0, β_0

$$q(\theta, \beta) = \frac{\delta(\theta - \theta_0)\,\delta(\beta - \beta_0)}{r_0^2 \sin\theta_0}, \quad f_{nm} = \frac{2\pi\varepsilon_m}{2n+1} \frac{(n+m)!}{(n-m)!} r_0^{-2} Y_n^{(m)}(\theta_0, \beta_0)$$

The substitution of (2. 2) and (2. 3) into boundary condition (2. 1) at the sphere surface $(r = r_0)$ yields

$$\sum_{n=0}^{\infty} \sum_{m=0}^{n} \left\{ A_{nm} \left[b(\lambda_{0n}^2 - \lambda^2) \frac{dh_n^{(2)}}{dr} + h_n^{(2)} \right] - f_{nm} \right\} \Delta_2 Y_n^{(m)} = 0$$

By equating to zero the coefficients at harmonics $Y_n^{(m)}$ $(n, m = 0, 1, 2, \ldots)$ we reduce this to a system of algebraic equations from which we obtain expressions for constants A_{nm}. Substituting these expressions in (2. 2) for A_{nm}, we obtain for the pressure distribution in the medium surrounding the oscillating spherical shell the formula

$$p(r, \theta, \beta) = \sum_{n=0}^{\infty} R_n(kr) \sum_{m=0}^{n} f_{nm} Y_n^{(m)}(\theta, \beta) \tag{2.4}$$

$$R_n(kr) = h_n^{(2)}(kr) \left[2h \frac{\rho_0}{\rho_f} \left(\frac{\omega_{0n}^2}{\omega^2} - 1 \right) \frac{dh_n^{(2)}(kr)}{dr} \Big|_{r=r_0} + h_n^{(2)}(kr_0) \right]^{-1}$$

Let us consider the radial pressure component $R_n(kr)$. After a number of identical transformations formula (2. 4) for R_n assumes the form

$$R_n(x) = \sqrt{\frac{x_0}{x}} \frac{J_q(x) - iN_q(x)}{\mathrm{Re}(f(x_0)) + i\,\mathrm{Im}(f(x_0))} \tag{2.5}$$

$$\mathrm{Im}(f(x_0)) = J_q(x_0)\,\mathrm{Im}(x_0) - N_q(x_0)\mathrm{Re}(x_0)$$

$$\mathrm{Re}(f(x_0)) = J_q(x_0)\,\mathrm{Re}(x_0) + N_q(x_0)\mathrm{Im}(x_0)$$

$$\mathrm{Im}(x_0) = g_n \frac{J_q(x_0) N_{q+1}(x_0) - J_{q+1}(x_0) N_q(x_0)}{J_q^2(x_0) + N_q^2(x_0)}$$

$$\mathrm{Re}(x_0) = g_n \left[\frac{n}{x_0} - \frac{J_q(x_0) J_{q+1}(x_0) + N_q(x_0) N_{q+1}(x_0)}{J_q^2(x_0) + N_q^2(x_0)} \right] + 1$$

$$g_n = 2 \frac{h}{r_0} \frac{\rho_0}{\rho_f} \left(\frac{\omega_{0n}^2}{\omega^2} - 1 \right) x_0, \quad x = kr, \quad x_0 = kr_0$$

Since solution (2. 4) is valid for considerable variations of the stress-strain state, it is possible to use asymptotic formulas for Bessel functions $J_q(x)$ and Neumann functions $N_q(x)$, whose form depends on the relation between the argument and the index of the corresponding cylindrical function [5]. We have the following asymptotic formulas: for $q = x \cos \alpha$ and large q

$$J_q (q \cos^{-1} \alpha) = \sqrt{2} (\pi q \operatorname{tg} \alpha)^{-1/2} \cos (q \operatorname{tg} \alpha - q\alpha - \pi / 4) + O(q^{-3/2})$$
$$N_q (q \cos^{-1} \alpha) = \sqrt{2} (\pi q \operatorname{tg} \alpha)^{-1/2} \sin (q \operatorname{tg} \alpha - q\alpha - \pi / 4) + O(q^{-3/2})$$

and for $q = x \operatorname{ch} \alpha$

$$J_q (q \operatorname{ch}^{-1} \alpha) = (2\pi q \operatorname{th} \alpha)^{-1/2} \exp [-q (\alpha - \operatorname{th} \alpha)] + O(q^{-3/2}) \tag{2.6}$$
$$N_q (q \operatorname{ch}^{-1} \alpha) = (1/_2 \pi q \operatorname{th} \alpha)^{-1/2} \exp [q (\alpha - \operatorname{th} \alpha)] + O(q^{-3/2})$$

Let us consider separately the integrals for which the condition $x < q$ is satisfied at points of the sphere and in its neighborhood. The asymptotic formula for $N_q (x)$ then defines a decreasing and for $J_q (x)$ an increasing component $h_n^{(2)} (x)$ with increasing x. For large q the inequality $N_q^2 (x_0) \gg J_q^2 (x_0)$ is satisfied on the sphere surface by virtue of the radiation condition which stipulates that functions N_q and J_q must be of the same order, as $r \to \infty$. These functions in approaching the shell reach the reversal point at $x = q$ where they change from oscillating at infinity to exponentially varying functions. As r further decreases, function J_q sharply decreases, while N_q sharply increases.

Taking into account the above inequality, we obtain for $R_n (x_0)$ the formula

$$R_n (x_0) = [\operatorname{Re} (x_0) - i \operatorname{Im} (x_0)]^{-1} \tag{2.7}$$
$$\operatorname{Re} (x_0) = g_n \left[\frac{n}{x_0} - \frac{N_{q+1} (x_0)}{N_q (x_0)} \right] + 1$$
$$\operatorname{Im} (x_0) = g_n N_q^{-2} (x_0) [J_q (x_0) N_{q+1} (x_0) - J_{q+1} (x_0) N_q (x_0)]$$

Substitution of the asymptotic formulas (2.6) into the real and imaginary parts of the denominator in (2.7) yields

$$\operatorname{Re} (x_0) = 1 - g_n (a_n \exp \gamma - n/x_0), \quad \gamma = (q + 1) (\alpha_1 - \operatorname{tg} \alpha_1) -$$
$$q (\alpha_0 - \operatorname{th} \alpha_0)$$
$$\operatorname{Im} (x_0) = a_n g_n \operatorname{sh} (q\gamma) \exp [-2q (\alpha_0 - \operatorname{th} \alpha_0)], \operatorname{ch} \alpha_1 = (q+1)/x_0$$
$$a_n = [q / (q + 1)]^{1/2} (\operatorname{th} \alpha_0 / \operatorname{th} \alpha_1)^{1/2}, \quad q = n + 1/_2,$$
$$n \gg 1, \quad \gamma > 0$$

This shows that the quantity $\operatorname{Im} (x_0)$ is asymptotically small (of order e^{-2q}) and decreases as the variation of the shell stress-strain state increases, but does not vanish for any finite n.

Presence of the small imaginary component in the denominator of the expression for pressure indicates radiation of a small part of energy of the oscillating shell to the fluid. The real part $\operatorname{Re} (x_0)$ of the denominator is at the same time of the asymptotic order of unity, but may vanish at some frequencies. When such frequencies coincide with that of the external excitation force, a sharp increase of pressure amplitude, an effect similar to resonance, takes place. Frequencies at which $\operatorname{Re} (x_0) = 0$ result in resonance; at such frequencies the pressure amplitude $| R_n (x_0) |$ is of order e^{2q}, while at nonresonant frequencies it is of order unity.

Let us investigate the behavior of the radial component of pressure (2.5) near the shell. Eliminating the imaginary part in the denominator in (2.5) we obtain:
in the resonance case

$$R_n(x) = \frac{(x_0/x)^{1/2}}{N_q^2(x_0)\,\mathrm{Im}(x_0)}[J_q(x)N_q(x_0) - N_q(x)J_q(x_0) - iN_q(x)N_q(x_0)]$$

away from resonance $(\mathrm{Re}(x_0) \gg \mathrm{Im}(x_0))$

$$R_n(x) = -\frac{(x_0/x)^{1/2}}{\mathrm{Re}(x_0)}\left\{\frac{N_q(x)}{N_q(x_0)} - i\left[J_q(x_0)\frac{N_q(x)}{N_q^2(x_0)} - \frac{J_q(x)}{N_q(x_0)} + \frac{N_q(x)}{N_q(x_0)}\frac{\mathrm{Im}(x_0)}{\mathrm{Re}(x_0)}\right]\right\}$$

Substituting the asymptotic expressions (2.6) for J_q and N_q and expanding the remainder $\alpha - \mathrm{th}\,\alpha$ in series in powers of $x - x_0$, we find that at resonance frequencies

$$R_n(x) = b_n x^{-1}\{A_p \exp[q(1 - x/x_0)\,\mathrm{th}\,\alpha_0 + O((x-x_0)^2)] + B_p \exp[-q(1 - x/x_0)\,\mathrm{th}\,\alpha_0 + O((x-x_0)^2)]\} + O(q^{-1}) \qquad (2.8)$$

$$A_p = i\exp[2q(\alpha_0 - \mathrm{th}\,\alpha_0)] + {}^1/_2, \quad B_p = -{}^1/_2, \quad b_n = \frac{x_0}{g_n\,\mathrm{sh}\,\gamma}\left(\frac{\mathrm{sh}\,\alpha_1}{\mathrm{sh}\,\alpha_0}\right)^{1/2}$$

and at frequencies away from resonance

$$R_n(x) = (x_0/x)\{A_H \exp[q(1 - x/x_0)\,\mathrm{th}\,\alpha_0 + O((x-x_0)^2)] + B_H \exp[-q(1 - x/x_0)\,\mathrm{th}\,\alpha_0 + O((x-x_0)^2)]\} + O(q^{-1}) \qquad (2.9)$$

$$A_H = 1 + i\,\mathrm{Re}^{-1}(x_0), \quad B_H = -{}^1/_2 \exp[-2q(\alpha_0 - \mathrm{th}\,\alpha_0)]$$

These formulas show that the absolute value of the coefficient at the exponentially increasing component of the solution is small in comparison with the coefficient at the exponentially decreasing component, and can be neglected. It should be, however, borne in mind that at infinity $x \gg q$ and both components are of the same order, as already pointed out.

Note that formulas for fading and increasing integrals can be obtained without resorting to asymptotic formulas. To show this we reduce Eqs. 2.1) for $R(r)$ to the form

$$r^2\Psi''(r) + [(kr)^2 - n(n+1)]\Psi(r) = 0, \quad \Psi(r) = rR(r)$$

whose approximate solution

$$R(r) = r^{-1}\exp\{\pm[n(n+1) - (kr_0)^2]^{1/2}(r/r_0)\} \qquad (2.10)$$

is valid near the sphere under condition that $(kr)^2 - n(n+1) < 0$. For large n this solution coincides either with (2.8) or (2.9) within the constants that are determined by comparison with the asymptotically exact solution.

3. The approximate solution (2.10) was used for calculating resonance oscillation frequencies of the spherical shell in fluid for the following values of parameters:

$$h/r_0 = 0.01, \quad \rho_f/\rho_0 = 0.13, \quad \nu = 0.3, \quad c_0 = 5\cdot 10^3 \ \mathrm{m/s}$$

The calculated dependence of resonance frequencies of the shell in fluid from the meridional wave number n appears in Fig. 1, where for comparison the similar dependence for resonance frequencies of a "dry" shell is shown by the dashed line. It will be seen that in the case of shell oscillations in fluid the mode with zero number of waves is absent along the meridian. The presence of that mode would violate the inequality $x < q$, i.e. the condition of appearance of this type of oscillations. When the variation [of frequency] is small, the resonance frequencies of the shell in fluid considerably lower than those of the dry shell. As the variation increases, the oscillation frequencies of the dry and the immersed in fluid shells tend to become equal, and for $n > 10$ the shell oscillates as if with a constant apparent mass of fluid added to it.

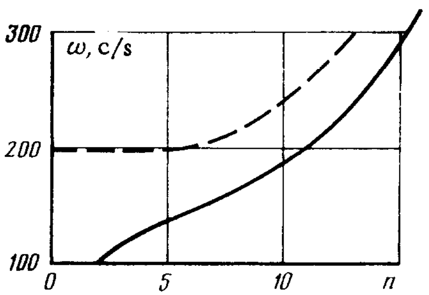

Fig. 1

The discrepancy between solution of the frequency equation $\operatorname{Re}(x_0) = 0$ by substituting into it the asymptotic expressions (2.6) and the approximate solution (2.10) was evaluated on the example of a specific shell. This showed that with $n = 5$ and greater, i.e. essentially from the bound of applicability of asymptotic formulas (2.6), the difference is less than 10% and diminishes as n is increased. At resonance frequencies the pressure amplitude increases to the level $O(e^{2q})$ $(q \gg 1)$, while in the nonresonance case it is of order unity. The calculated ratios of pressure amplitudes at resonance frequencies to those in the non resonance case are: 9.775, 18.336, 39.923, 86.52, 182.43 for the first meridional numbers $(n = 1, 2, \ldots, 5)$. This shows that damping of resonance frequencies of the shell in fluid owing to energy radiation is very small, and is more apparent when the variation of the stress-strain state is small.

4. Solution (2.10) which shows the pattern of pressure damping in a medium with an oscillating spherical shell will be used here as the standard in investigations of oscillations of an arbitrary closed convex shell in fluid. We limit the investigation to high-frequency oscillations for which the inequality [2]

$$\lambda^2 \gg \max(R_1^{-2}, R_2^{-2}) \tag{4.1}$$

is valid, and Eqs. (1.1) for the shell oscillation pattern reduce to the single equation

$$h_*^2 \Delta_2^2 W - \lambda^2 W + p\,|_S - q = 0 \tag{4.2}$$

We introduce in the neighborhood of the shell the orthogonal system of coordinates (α, β, z), where α and β are isothermal coordinates that coincide with curvatures of the shell median surface, and z is the external normal to the surface. In these coordinates the first quadratic form is

$$ds^2 = H^2(\alpha, \beta)\,([1 + zR_1^{-1}(\alpha, \beta)]^2 d\alpha^2 + [1 + zR_2^{-1}(\alpha, \beta)]^2 d\beta^2) + dz^2$$

where $H(\alpha, \beta)$ is the Lamé parameter of the isothermal system (α, β).

By analogy to (2.10) we assume that pressure in the neighborhood of the shell varies in conformity with the law

$$p(\alpha, \beta, z) = F(\alpha, \beta)[r_0(\alpha, \beta) + z]^{-1} \exp(-az) \tag{4.3}$$

in which $r_0^{-1} = (R_1 R_2)^{-1/2}$ is the normal curvature of the surface at point (α, β), and $a > 0$ is an unknown large constant $(a = O(k))$ which defines the pressure damping rate. Taking (4.3) as the input equation, and taking into account the relation between the deflection and normal pressure derivative (1.2) at the surface and, also, inequality (4.3), we obtain that

$$p(\alpha, \beta, 0) = -W(\alpha, \beta) / (ab) \tag{4.4}$$

As shown above, the resonance oscillation frequencies of the shell in fluid are reasonably exactly determined when only the damped pressure component is taken into account (allowance for the increasing component is only necessary for the determination of pressure amplitude). The respective particular equation is independent of the method of shell loading, hence it is possible to set $q = 0$ when determining resonance frequencies. Equation (4.2) with allowance for (4.4) then assumes the form

$$\Delta_2^2 W - \Omega^4 W = 0, \quad \Omega^4 = h_*^{-2}(\lambda^2 + 1/(ab)) \tag{4.5}$$

Equations of this type were considered in [6]. For closed shells the solution of (4.5) is equivalent to the solution of the Helmholtz equation

$$\Delta_2 W + \Omega^2 W = 0 \tag{4.6}$$

When variations of the stress-strain state are considerable, the integrals of this equation are concentrated in the neighborhood of equatorial geodetic lines of the shell median surface and are of the form

$$W = D_q(\sqrt{2\mu}\Psi)e^{i\mu\Phi}$$

where Ψ and Φ are functions of coordinates and of the large parameter μ, $q = 0, 1, 2, \ldots$ is the meriodional wave number, and $D_q(\sqrt{2}t)$ is the Hermite function oscillating within band between two transition lines when $|t| \leqslant \sqrt{2q+1}$ exponentially damping outside that band. Omitting intermediate operations for the determination of μ, Ψ and Φ which appear in [6], we write the final asymptotic formula for the frequency parameter

$$\Omega_{q,m} = 2\pi m \left[\int_0^L H(0, \beta)\, d\beta\right]^{-1} \left[1 - \frac{2q+1}{4\pi m} \int_0^L \frac{d\beta}{F_*^2(\beta)} + O\left(\frac{1}{m^2}\right)\right] \tag{4.7}$$

$$(q = 0, 1, 2, \ldots)$$

in which $m \gg 1$ is the wave number along the equator, β is the current arc of the equator of length L, and $F_*(\beta)$ is a function representing the real periodic along the equator solution of the equation considered in [7].

The parameter $\Omega_{q,m}$ is related to the resonance oscillation frequency $\omega_{q,m}$ by formula (4.5) which contains the unknown constant a. To determine a we use the Helmholtz equation for pressure at the shell surface (for $z = 0$)

$$\Delta_2 p + (R_1^{-1} + R_2^{-1})\partial p / \partial z + \partial^2 p / \partial z^2 + k^2 p = 0$$

in which we substitute expression (4.3) and, taking into account (4.1) and (4.6) obtain $a = (\Omega^2 - k^2)^{1/2}$. The obtained expression for the damping coefficient is substituted into (4.5). After a number of identical transformations we obtain for the real part of oscillation frequency of the shell in fluid the equation

$$\mu^3 + (e_0^2 - e - 2)\mu^2 + (2e + 1)\mu - e = 0, \quad \mu = \omega^2 / \omega_0^2$$
$$e = \frac{c_f^2}{c_0 h_* \omega_0}, \quad e_0 = \frac{1}{2h}\frac{\rho_f}{\rho_0}\frac{c_f}{\omega_0}, \quad \omega_0 = c_0 h_* \Omega$$

For a numerical evaluation of the last formula applicability region it was compared in the case of a spherical shell with the analogous asymptotically exact formula for the damping coefficient in the index of the exponent of solution (2.8). This showed that for $n > 8$ the two formula differed by less than 10%.

Thus formulas (4.5), (4.7), and (4.8) make possible the determination of resonant frequencies of high-frequency oscillations of a closed convex shell of arbitrary form immersed in a compressible fluid, and, also, the rate of acoustic pressure damping with increasing distance from the shell.

REFERENCES

1. T s i o n s k i i, A. Ia., Oscillations of a spherical shell in a compressible fluid. Izv. Sev. - Kavkazsk. Nauchn. Tsentra Vysshei Shkoly. Ser. Estestv. Nauk, No. 1, 1977.

2. G o l ' d e n v e i z e r, A. L., Classification of integrals of the dynamic equations of the linear two-dimensional theory of shells. PMM, Vol. 37, No. 4, 1973.

3. M o r s e, F. M., and F e s h b a k h, H. Methods of Theoretical Physics, McGraw-Hill, 1953.

4. T i k h o n o v, A. N. and S a m a r s k i i, A. A., Equations of Mathematical Physics. Moscow, "Nauka", 1966.

5. W a t s o n, G. N., Theory of Bessel Functions, pt. 1, Cambridge, C.U.P. 1948.

6. P o p o v, A. L., Asymptotics of eigenfunctions and of frequencies of short-wave oscillations of a closed shell. Izv. Akad, Nauk SSSR, MTT, No. 2, 1977.

7. B a b i c h, V. M. and B u l d y r e v, V. S., Asymptotic Methods in Problems of Short-wave Diffraction. Moscow, "Nauka", 1972.

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